Convection laws for glass furnaces revisited

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We introduce a simple geometry to study natural convection in a two-dimensional box, closer to that of glass furnaces than the usual differentially heated box. We show that, in both cases, temperature and velocity fields scale with the Rayleigh number (or the Grashof number) based on the height of the box. The model we suggest has qualitative features similar to those observed in direct numerical simulations of glass furnaces.

Introduction

Convection is the most remarkable large scale phenomenon occurring in glass furnaces. Its role is essential to insure proper melting and to achieve the required levels of quality. Its intensity largely depends on the geometric configuration of the furnaces. Natural convection has been intensively studied by physicists, theoretically as well as experimentally. Despite this body of knowledge, glass makers hardly quote academic literature on that topic, the most accessible reference being Leyens' analysis[1] as reported in Trier[2]. In this paper, we first summarize results on differentially heated long cavities and we discuss the consequences for glass furnaces. We then present results on convection in a zenithally inhomogeneously heated long cavity, a geometry closer to that of glass furnaces.

The differentially heated cavity

G. Leyens' work to quantify the intensity of convection in a glass furnace relies on a comparison with an academic convection problem, namely convection in a long adiabatic cavity with differentially heated end walls. Despite numerous studies on this topic, knowledge is far from complete, especially for convection at high Rayleigh number and high Prandtl number. We first recall the main known results as summarized by B. Boehrer[3]. We then describe how they may be recovered in the simplest case and compare them to glass furnaces.

Various possible regimes

We consider a long cavity of aspect ratio $A \equiv H/L$ ($A << 1$) with adiabatic upper and lower walls, and with side walls kept at fixed temperatures T_- and T_+ (Fig. 1a). The fluid which fills the cavity has constant kinematic viscosity ν and heat diffusivity κ. Its density ρ varies linearly with temperature, the thermal expansion coefficient being β. It is a classical matter to show that this problem only depends on three nondimensional numbers: the Rayleigh number $Ra = g\beta\Delta T H^3/(\nu\kappa)$, the Prandtl number $\Pr = \nu/\kappa$ and the aspect ratio A. For glass, heat diffusivity is usually dominated by the radiative component within the framework of the Rosseland approximation. Also, viscosity typically varies over two to three orders of magnitude in the melting part of a furnace. However, the Prandtl number remains high, of order 10^2 to 10^3. Assuming typical values for glass and furnace characteristics, the Rayleigh number is of order 10^6 to 10^7, and the aspect ratio is about $5 \cdot 10^{-2}$. B. Boehrer[3] has summarized known features of convection in a long box, at high Rayleigh number and high Prandtl number. Three different regimes exist : the conductive, the transition and the convective regime, the boundaries of which he discusses. In the

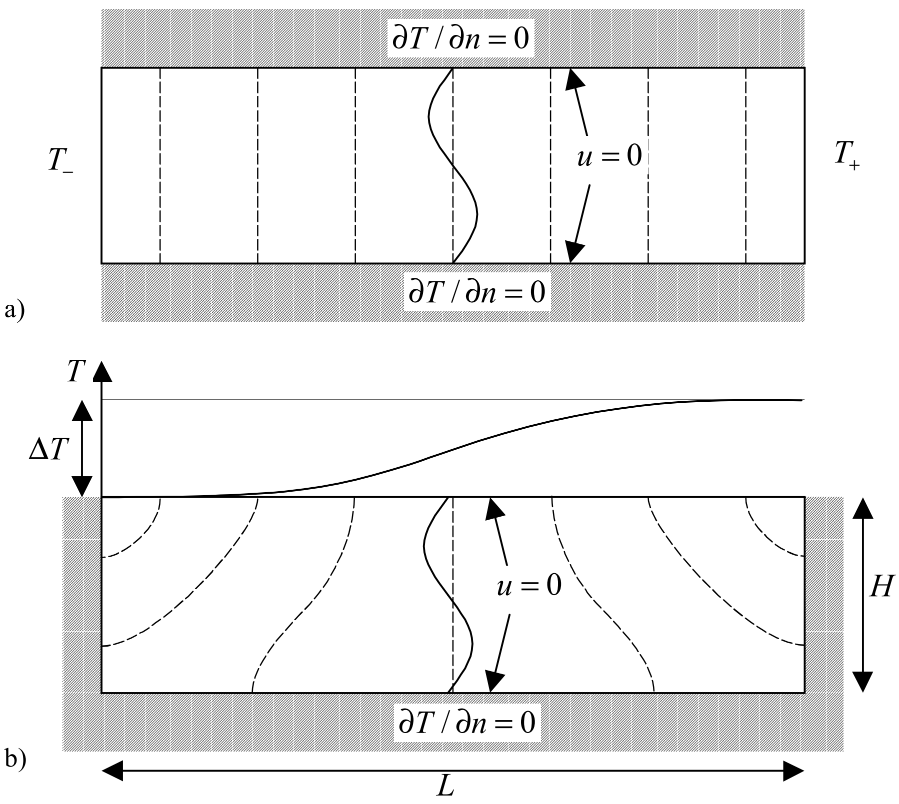

Figure 1. Model cavities for natural convection. a) Differentially heated cavity. b) Zenithally heated cavity. Isotherms (dashed lines) and velocity profile (solid line) are sketched in the conductive regime.

conductive regime, isotherms are essentially vertical and the temperature gradient is roughly constant in the bulk of the cavity. In the convective regime, the center of the cavity is essentially motionless, the whole transport taking place in two boundary layers at the top and the bottom of the cavity ; consequently, the isotherms are mainly horizontal. In the transition regime, the isotherms flatten out whereas the fluid moves in the whole cavity.

With phenomenological arguments relying on experimental results, Boehrer suggests that the transition regime occurs when $Ra.A^2$ is between 10^2 and 10^4. Hence inasmuch as glass furnaces may be compared to this ideal situation, the regimes of interest are the conductive and the transition one. Thus results obtained in the conductive regime will be appropriate for, or close to, glass furnaces.

Conductive regime

The structure of the velocity and temperature fields are the simplest in the conductive regime. Since the temperature gradient is constant (of order $\Delta T / L$), and the flow parallel (except near the side walls), the pressure field is hydrostatic with a temperature dependent density. The equation of motion in the bulk reduces to a force balance along the longitudinal direction : $\frac{\partial^2 u}{\partial z^2} = -\frac{\beta g}{\nu}(H - z)\frac{\partial T}{\partial x}$, which is easily integrated. The maximum velocity reads : $u = 8 \cdot 10^{-3} Ra(\kappa / L) = 8 \cdot 10^{-3} Gr(\nu / L)$, where Gr is the Grashof number ($Gr = Ra/\Pr$). This simple analysis is confirmed by asymptotic analysis[4].

Comparison with classical formulae

Trier, quoting Leyens, explains that the Rayleigh number, as well as the Grashof number can be used as similarity parameters. However he does not precise how different quantities scale. In addition, he suggests that the product $H^2.L$ should be used in the Rayleigh (or the Grashof) number. According to our analysis, this seems inappropriate.

The zenithally heated cavity

Since the differentially heated long cavity is still far from resembling a glass furnace, we suggest a similar model, simple enough for calculation, but somewhat closer to reality. We consider a closed box, with adiabatic side walls and bottom, with a half-cosine imposed temperature profile along the top side (Fig. 1b).

Temperature field without flow

In the conductive regime, since the Prandtl number is high, the velocity field adjusts to the temperature field. As a first approximation, the latter can be evaluated with the static field, which is a solution of the Laplace equation : $\nabla^2 T = 0$, in the bulk, with the appropriate boundary conditions ($\partial T / \partial n = 0$ along the adiabatic sides, $T = \overline{T} - (\Delta T / 2)\cos(\pi x / L)$ along the top side). This can be calculated by Fourier transform and separation of variables. One finds for the dimensionless temperature θ : $\theta_0(\hat{x},\hat{y}) = \frac{1}{2}\left[1 - \frac{\cosh \pi A \hat{y}}{\cosh \pi A}\cos \pi A \hat{x}\right]$, where the horizontal and vertical coordinates x and y have been normalized by the height H of the box ($\hat{x} = x / H$, $\hat{y} = y / H$). The structure of the isotherms is richer in this case than in the differentially heated cavity : they symmetrically fan out from the center of the cavity (Fig. 1b).

Structure of the flow

As expected, the perturbation induced by the velocity field to the temperature field is small at low Rayleigh number. Asymptotic analysis can be carried out and leads to the following result at the lowest order in Ra :

$$\theta(\hat{x},\hat{y}) \approx \frac{1}{2}(1 - \cos \pi A \hat{x}) - \frac{A^2\pi^2}{4}(\hat{y}^2 - 1)\cos \pi A \hat{x} + Ra\frac{A^2\pi^2}{4}\left[\frac{\sin^2 \pi A \hat{x}}{24}(\frac{\hat{y}^5}{5} - \frac{\hat{y}^4}{2} + \frac{\hat{y}^3}{3} - \frac{1}{30})\right]$$

At zeroth order in Ra, the stream function reads : $\psi_0(\hat{x},\hat{y}) = \frac{\pi}{48}\hat{y}^2(\hat{y} - 1)^2 \sin \pi A \hat{x}$. These formulae indicate some simple results. First of all, comparison between asymptotic formulae show that the scaling is similar to that seen in the differentially heated cavity. In particular, the maximum velocity also scales like $Ra(\kappa / L)$. One also sees that the relevant parameter to classify the different regimes is also $Ra.A^2$. Finally, with increasing Ra, because of the interplay between velocity and temperature (in other words, because of convection),

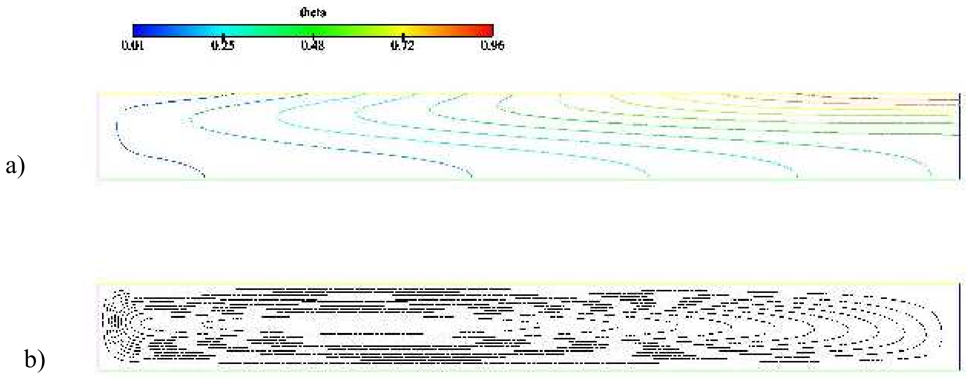

Figure 2. Numerical calculation of zenithally heated cavity. ($Ra=10^5$, $A=0.1$, $Pr=10^3$).
a) Isotherms. b) Streamlines.

the temperature field looses its left/right symmetry. Isotherms tend to be vertical in the cold place (linear thermal gradient) and to be horizontal in the hot place (stably stratified). This tendency is commonly observed in glass furnaces simulations[5]. As expected, these formulae loose their validity when the Rayleigh number becomes too large, but the main features remain (Fig. 2).

Conclusion

Comparison of two simple models of natural convection in a long cavity shows that the relevant scaling parameter is the Rayleigh (or the Grashof) number based on the height of the cavity only. In particular the maximum bulk velocity is proportional to H^3/L. A modified version of the usual differentially heated cavity, namely the inhomogeneously zenithally heated cavity shows qualitative features similar to those observed in direct simulations of glass furnaces. It would be interesting to test the impact of variations of physical properties of the fluid in this configuration.

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