Analysis of fracture features for uni-axial bent glass.plates

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Glass plates of different thickness are fractured in a uniaxial bending test under a variety of conditions: different surface damages, and rates of applied stress are considered. The fracture stresses, the crack lengths prior to branching and the sizes of the mirror are measured. As expected, the experimental results show a correlation between the fracture stress and these fracture features. They also reveal the existence of a possible threshold stress below which no mirror - mist boundary nor branching is found for bending fracture. An analysis is proposed, based on the elastic energy stored in the tensile stress region. The analysis explains the existence of a threshold stress, which depends on the glass thickness.

I. Introduction

As described by J.B. Quinn, and D.K. Shetty [1,2], there exists an empirical relation between the stress at fracture (σ_r) and the mirror radius (r_m) or the length at branching (r_b):

$$\sigma_r = \frac{A_i}{\sqrt{r_i}} + \sigma_{0,i} \text{, i = m, or b} \qquad (1)$$

Although the mirror or branching length constants A_i are not intrinsic properties of the material, values reported in the literature are around A_i = 2 MPa.m$^{1/2}$, whatever the glass surface quality, the environment, the loading rate, or the loading geometry (pure traction, uniaxial or biaxial bending). This value is roughly 2.7 times that of the toughness of the glass. The origin stress constants $\sigma_{0,i}$ are function of the loading geometry. In pure traction, $\sigma_{0,i} = 0$ [2], in biaxial bending, $\sigma_{0,i} > 0$ [1]. There also seems to be another test dependent constant, the threshold stress $\sigma_{s,i}$. It can defined as the smallest stress at fracture, for which a finite mirror radius or branching length is observed. However, in the literature, no explanation is given to justify the existence of such stresses. The objective of this paper is to study this phenomenon and to propose an interpretation.

II. Experimental Procedure

The objectives of the experimental study is to determine the effect of the glass thickness on the fractography parameters. Two series of soda-lime glass samples are investigated. The dimensions of the glass plate are 300 mm × 120 mm. The first series are 7.88 mm thick, annealed glass plates. The second series are 2.15 mm thick, annealed glass plates. In both cases, the measured tensile residual core stress σ_c is smaller than +0.30 MPa.

Different controlled defects are used in order to obtain a wide range of fracture stresses : Vickers static indentations (the load varies from 0.5 to 10 daN), dynamic indentations (the diamond weight is 8.2 g, the dropping height varies from 0.2 to 1.3 m), and cutting defects.

The four point bending test is used for the study. The applied stress is uni-axial. The distance between the supporting rollers is 250 mm, and 150 mm between the loading rollers. For each thickness, the relation between the applied stress and the load is established once using strain gages. The different loading rates used are: 4.5 MPa/s, 4.5 10^{-2} MPa/s, and 4.5 10^{-3} MPa/s. The environment conditions are not controlled. In each case, the fracture stress, the mirror radius, and the branching length are recorded.

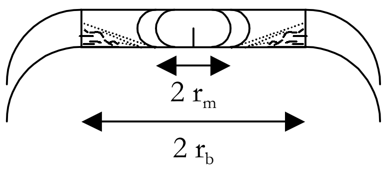

Figure 1.Oblique schematic representation of measured features

III. Results

Figures 2 and 3 are graphs of the fracture stress versus the reciprocal square root of the mirror radius and the branching length.

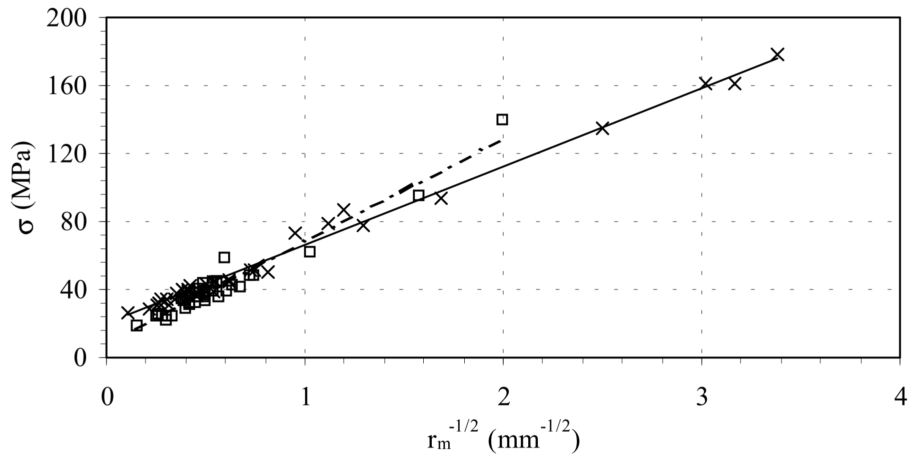

Figure 2. Fracture stress versus reciprocal square root of the mirror radius.
(: e = 7.88 mm, $A_m = 60.873\ mm^{-1/2}$, $\sigma_{0,m} = 6.88$ MPa ;
×: e = 2.15 mm, $A_m = 46.413\ mm^{-1/2}$, $\sigma_{0,m} = 19.54$ MPa)

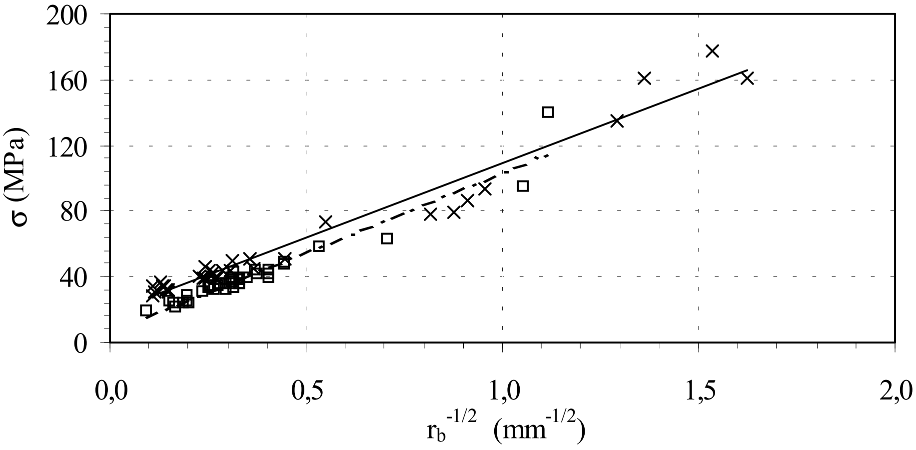

Figure 3. Fracture stress versus reciprocal square root of half the crack length at branching. (: e = 7.88 mm, A_b = 98.039 $mm^{-1/2}$, $\sigma_{0,b}$ = 5.67 MPa ;
×: e = 2.15 mm, A_b = 91.03 $mm^{-1/2}$, $\sigma_{0,b}$ = 17.95 MPa)

IV. Modeling

A rectilinear crack front through the glass thickness is considered. It is treated in the case of a uni-axial bending stress, self equilibrated through the thickness. We assume that the propagation of the crack is controlled by the stress intensity factor (SIF) at the intersection of the crack front with the glass surface, where the tensile stress is maximum (K_{max}). This SIF is calculated numerically from the contour integral J [3], taking into account the crack closure in the compression zone, using the Abaqus software. The SIF can be written as a function of the maximal tensile stress, and the crack length (a) by introducing a shape factor (Y_p, the index p stands for the kind of stress profile through the glass thickness). Y_p is a function of the plate thickness to crack length ratio (e/a), and the glass Poisson coefficient (ν). Y_p is calculated numerically from equation (2), and plotted in figure 4, in the case of uni-axial bending.

$$K_{\max} = Y_p(e/a,\upsilon)\sigma\sqrt{a} \quad (2)$$

In figure 4, we note that, when the crack length is large in front of the glass thickness, for example for (e/a)<0.65 [4], the shape factor can be written in the form:

$$Y = \alpha\sqrt{e/a} \quad (3)$$

For the uni-axial bending case, we find α = 0.63. Equations (2) & (3) lead then to equation (4).

$$K_{\max} = \alpha \times \sigma \times \sqrt{e} \quad (4)$$

The elastic energy stored in the tensile zone for a uni-axial bent plate can be scaled with :

$$\int_{\text{tensile zone}} \sigma^2(z)dz = \sigma^2 e/6 \qquad (5)$$

This shows that, for a long (through the thickness) crack in a uni-axially bent plate, the evaluation of the SIF from the energy stored in the tensile zone is valid. This validity has already been shown in the case of plates with a symetric residual stress field, in thermal and chemical tempering [5].

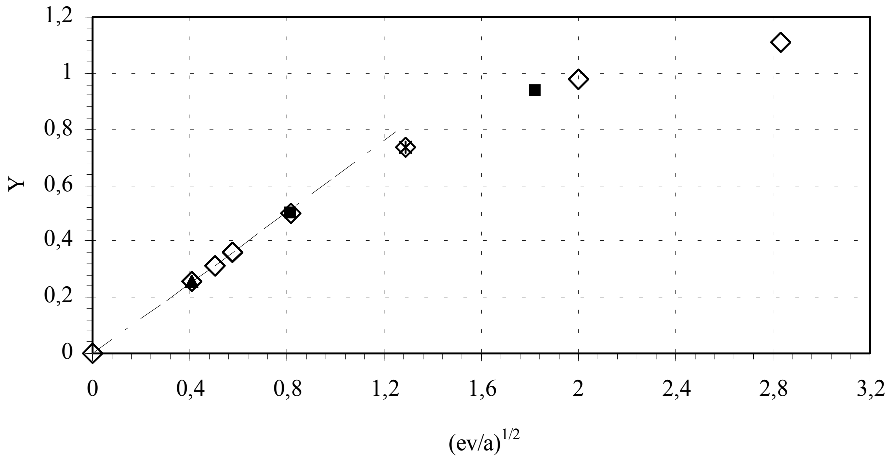

Figure 4. Shape factor as a function of the square root of e/a, calculated numerically for a crack under different bi-axial stress field, for different glass plate thickness. (◇: σ = 50 MPa, e = 2 mm; ▲ σ = 100 MPa, e = 2 mm; ■: σ = 50 MPa, e = 4 mm, * : σ = 35 MPa, e = 4 mm).

V. Discussion

Experimental data of figures 2 & 3 are more scattered than that of J.B.Quinn for bi-axial bending [1]. Given this scattering, the A_i values of equation (1) seem to be roughly independent of glass thickness : $A_m \sim 1.7$ MPa.m$^{1/2}$, and $A_b \sim 3$ MPa.m$^{1/2}$. From ([3]), $A_m = A_b \sim 2$ MPa.m$^{1/2}$. On the contrary, the stresses at the origin ($\sigma_{0,i}$) and the threshold stresses ($\sigma_{s,i}$) seem to depend on the glass thickness. The higher the glass thickness, the lower they are. This is not surprising, if we look at the modeling results. Indeed, we suggest that $\sigma_{0,i}$ is the lowest bending stress which leads to fracture without any mist zone or bifurcation, and $\sigma_{s,i}$ is the lowest bending stress which leads to fracture with a mist zone and a bifurcation. Each case corresponds respectively to $K_{max} = K_c$, and $K_{max}=K_m$ or K_b, where K_c is the glass toughness, and K_m and K_b the SIF corresponding to the mirror-mist transition and the bifurcation. In the case of soda-lime glass, $K_c = 0.76$ MPa$^{1/2}$, $K_m = 1.52$ MPa$^{1/2}$, $K_b = 2.7$ MPa$^{1/2}$ [6]. Such stresses can be calculated from equation (4). The calculated values overestimate the experimental ones (table 1). One reason might be that the core residual stress (σ_c), or the membrane stress (σ_a) of the bending test itself have not been considered in the calculation of K_{max}. One last suggestion would then be to modify equation (4) into equation (6). This modification is not treated in this present paper.

		Mirror		Bifurcation	
e (mm)		2.15	7.88	2.15	7.88
$\sigma_{0,i}$ (MPa)	experimental	19.5	6.9	18.0	5.7
	calculated from eq. (4)	26.0	13.6	26.0	13.6
$\sigma_{s,i}$ (MPa)	experimental	25.9	18.5	36.5	24.7
	calculated from eq. (4)	52.0	27.2	92.4	48.3

Table 1. Experimental and calculated $\sigma_{0,i}$ & $\sigma_{s,i}$ (K_c= 0.76, K_m= 1.52, K_b= 2.7MPa.m$^{1/2}$).

$$K = \alpha\sigma\sqrt{e} + \beta\sigma_c\sqrt{e} + \gamma\sigma_a\sqrt{\pi a} \quad (6)$$

with $\alpha = 0.63$, $\beta = 0.85$ ([5]), and for example $\gamma=1$ [7].

VI. Conclusion

It is shown that, in a uni-axial bending test, the sizes of the mirror and the crack branch follow the same kind of relationship as described by J.B.Quinn, in a bi-axial test [1]. The parameters in this relation depend on the glass thickness. The modeling of the SIF of a crack in a plate is suggested to explain this dependency. At the rectilinear front of a long crack through the glass thickness, the SIF depends only on the bending stress and on the square root of the glass thickness. It is verified that this SIF can also be estimated analytically using the elastic energy stored in the tensile zone.

[1] J.B.Quinn, JACS **82 [8]**, p. 2126-32 (1999).

[2] D.K.Shetty, JACS **Comm.**, p. C-10,C-12 (1983).

[3] H.D. Bui, in *Mécanique de la rupture fragile*, edited by Masson (1978).

[4] E. Le Bras in *XIV Intl. Congr. on Glass,* 1986, p. 262-269.

[5] O. Gaume in *XIX Intl. Congr. on Glass,* 2001, p. 106-107.

[6] F.Kerkhof, H. Richter in *2nd Int. Conf. of Fract.,* edited by Chapman & Hall, 1969, p. 463-473.

[7] B. Lawn, in *Fracture of brittle solids*, edited by E.A. Davis (Cambridge Solid State Science Series,1975, 1993).