Heat transfer in fibrous insulators.

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We are going to present a study about the modeling and the thermal characterization of semi-transparent media made of silica fibers. We shall be particularly interested in the radiatif heat transfer and in the interactions between the radiation and the material. The transmission and the reflection of the radiation are measured on commercial devices working with specific devices developed by the LEMTA. The aim of this studies is to improve the performances of building heat insulators and of materials used under strong thermic gradients. Typical example are resistance to fire, spatial engines materials behaviour. Transient nature of combined heat transfer by radiation and conduction in a fibrous medium is studied with temperature or flux boundary conditions. The monochromatic scattering and absorption coefficients as well as the phase function are calculated using the Mie theory. The equations modelling heat transfers are solved using a unidimensional model with numerical methods based on a discretization of the medium. Simulation results are presented for an insulator layer made of silica fibers. The study presented here concerns the characterization of fibrous insulations, but theory and experimental apparatus can be used for all the semi-transparent media.

Analysis

Introduction

The improvement of the thermal properties of a material needs a good understanding of the physical phenomena responsible of heat transfer. The studied materials are made of two phases: silica fibers and air (situated between fibers).

At room temperature, we can neglect the convection inside the medium: we shall have to consider only the transfers of heat by conduction and radiation. Heat conduction of insulators made the object of numerous studies at Saint-Gobain research center.

Conduction in fibrous media is due to conduction in air and conduction throught the fibers in contact. We are going to present here the methods used for the study of the radiative transfer. The size of particles constituting the medium is almost the same as radiation wavelength. So, the classic geometric optic laws are not sufficient to study the radiation-fiber interaction. The first step of our research is to solve the Maxwell equations.

We assume that a fiber can be represented by an infinitely long cylinder and we determine its radiative properties using the Mie theory [1-2-3]. These calculations are true if we set that fibers are sufficiently spaced from each others.

The equations of the radiative transfer and of energy conservation are solved with a method which allows the calculation of temperature and intensity fields in the medium. Radiative and conductive heat fluxes will be presented further.

In steady-state regime, our model has been validated by results obtained by Saint-Gobain, for several types of materials, with a hot guarded plate apparatus. The material is isolated from the outside and placed between two plates with fixed temperatures. According to the insulating quality of the material, it is necessary to bring more or less of heat (by Joule effect) to keep the hot face temperature constant. The amount of heat provided characterizes the insulating performance of the tested material.

The interest of this research is the optimization of the thermal properties of the fibrous medium from the knowledge of easily measurable properties.

The radiation–conduction interaction inside a semi–transparent medium is completely described by two equations. One express the radiative transfer and the other the total energy conservation. The fibrous media studied are non–grey, anisotropically absorbing, emitting and scattering. The radiative properties of the medium are characterized by the monochromatic absorption, scattering coefficients κ_λ, $\sigma_{s\lambda}$ and by the monochromatic phase function Φ_λ, that we determine from the Mie theory [1,2,3]. The thermophysical material properties (specific heat capacity and thermal conductivity) are temperature dependent. The medium is one dimensional, axisymetric with thickness *E*.

A. The Radiative Transfer Equation (RTE)

In such medium and for the applications we consider, the monochromatic radiation intensity $I_\lambda(x,\mu,t)$ is governed by the RTE [4], which is written for the wavelength λ, the position *x*, in the direction μ and at time *t* as

$$\mu \, \frac{\partial I_\lambda(x,\mu,t)}{\partial x} = -\,\beta_\lambda(\mu)\, I_\lambda(x,\mu,t) + J_\lambda(x,\mu,t) \tag{1}$$

$$J_\lambda(x,\mu,t) = \kappa_\lambda(\mu)\, I_{b,\lambda}(T(x,t)) + \frac{1}{2} \int_{\mu'=-1}^{1} \sigma_{S\lambda}(\mu')\, \Phi_\lambda(\mu' \to \mu)\, I_\lambda(x,\mu',t)\; d\mu' \tag{2}$$

The total radiative heat flux is given by

$$Q_r(x,t) = 2\pi \int_{\lambda=0}^{\infty} \int_{\mu=-1}^{1} I_\lambda(x,\mu,t)\, \mu \;\; d\mu \;\, d\lambda \tag{3}$$

$I_{b,\lambda}(T(x,t))$ is the intensity of the black body at the temperature of the considered point. $\sigma_{S\lambda}(\mu)$ and $\kappa_\lambda(\mu)$ are the effective section in the direction μ refer respectively to the absorption and scattering.

The second member of this RTE involves three terms linked to the phenomena taken into account for the diminution and the intensification of the intensity:

- The intensity can be decreased by extinction: scattering and absorption;
- The internal emission increases the intensity (it depends on the temperature field);
- By scattering, the radiative flux in the initial direction μ’ can increase the flux in the direction μ

B. The energy equation

The transient temperature response in the medium is obtained solving the nonlinear energy equation which is written at the position *x* and at time *t*, according to

$$\rho\, c_p \frac{\partial T}{\partial t}(x,t) - \frac{\partial}{\partial x}\left(k(T(x,t))\, \frac{\partial T}{\partial x}(x,t)\right) = S_r(x,t) \tag{4}$$

In our case, the medium thermal conductivity $k(T)$ is described by Langlais and Klarsfeld semi–empirical relation developed for insulators made of silica fibers. It is based on experimental data obtained from a guarded hot plates (apparatus at the Saint Gobain Research Center) [5]

$$k(T) = 0.2572\ T^{0.81} + \ 0.0527\ \rho^{0.91}\ (1 + 0.0013\ T) \tag{5}$$

The equation (4) is coupled to the radiative transfer through the radiative source term

$$S_r(x,t) = -\frac{\partial Q_r}{\partial x}(x,t) \tag{6}$$

where Q_r is given by the relation (3). The conductive heat flux is defined by

$$Q_c(x,t) = -\,k(T(x,t))\,\frac{\partial T}{\partial x}(x,t) \tag{7}$$

The total heat flux is given by the sum of radiative and conductive heat fluxes

$$Q_t\ = Q_r + Q_c \tag{8}$$

Under steady–state condition, the nonlinear energy equation (4) becomes

$$\frac{d\ Q_t}{dx}(x) = 0 \tag{9}$$

Determination of the optical index of the silica

The expression of the various coefficients occurring in the equation of the radiative transfer arises from the theory on the radiation - fiber interaction. So one needs the optical properties of the bulk material.

These properties are defined by the real part n_λ and imaginary part k_λ of the complex optical index given by:

$$n^*_\lambda = n_\lambda - ik_\lambda \tag{10}$$

n_λ is called index of refraction and k_λ characterises the absorption within our media. The index n_λ and k_λ were determinated from reflexion spectrum measurements using of Kramers-Krönig transformation. Transmission measurements are coupled with this method when the index k_λ becomes very small with respect to the index n_λ.

These measures are achieved at the LEMTA in a wide range of wavelengths: from the ultraviolet to far infrared. The numerical results obtained by this method allow us to represent the variations of the indications n_λ and k_λ according to the wavelength.

We observe in the figure 1 that the optical properties of the silica strongly depend on the wavelength. So the approximation of the grey material, i.e. a material the optical properties of which are independent of the wavelength, would be completely unjustified in this case. Moreover, analysis of the figure 1 shows the presence of two particular points.

They are located at wavelengths λ_1=7.3µm and λ_2=18.9µm. For these two wavelengths, the medium has the same refractive index n_λ as air, and the index k_λ is small, i.e., the absorption is weak. The medium made of fibers and air behaves like a no scattering and no absorbing material: it is the CHRISTIANSEN effect. The presence of these two CHRISTIANSEN filters must thus cause a decrease of the extinction of the radiation as we will show further.

The figure 1 shows also the presence of three absorption peaks, for the wavelengths 9.17µm, 12.5µm and 22µm, which can be related to movements of active ions by infrared illumination of the same frequency.

Radiative properties of the medium

Once the bulk material optical properties has been determined, one have to obtain the radiative properties of the fibrous medium that is an inhomogeneous medium made of similar "particles". In the independent diffusion approach, the scattering by the medium is equal to the sum of scattering by each particle and then the radiative properties of the medium i.e. the effective sections κ, σ_a and the phase function $\Phi_\lambda(\mu' \rightarrow \mu)$ are obtained by an average of single particle properties along the size distribution and orientation of the particles in the medium.

For a very few simple cases (sphere, ellipsoid and cylinder), the scattering by the particles can be obtained by the Mie approach which leads to nearly analytical results which involve series of functions such as Bessel functions or Ricati-Bessel functions [6]. In other case, one has to use numerical methods such as the use of the Green function and the Dyson equation [7], the development of the field on a multipole basis [8] or the use of the finite difference time domain method which is based on a direct resolution of Maxwell equations by using finite differences in the time domain [9]. The most important advantage of this last method is that we can obtain results in a wide range of wavelengths with no more than a Fourier transform of the temporal results that are obtained. It is then possible to gain a large amount of calculation time against the other methods which are monochromatic ones.

Here the silica fibers can be considered as infinitly long cylinders and the interaction with the radiation is known. We present in figure 2 variations of κ, σ_s versus the wavelength for a medium made of silica fibers, with a 4µm diameter, randomly oriented in planes parallel to the sample edges.

In some cases, the medium is so much complex that a direct determination of the radiative properties of the medium would be a herculean task. Then it would be easyier to obtain these radiative properties from experimental measures such as the bidirectional reflection or transmission functions (BRDF or BTDF), proceeding in this way by an inverse method [these arnaud ou articles acceptés]. For doing this, one has to obtain the effective section σ_e et σ_a (or in an equivalent way the the optical thickness and the albedo) and the parameters of a simplified phase function model [10] that lead, once used in the radiative transfer equation, to a minimal departure with experimental measures, that leads to an optimization problem.

Application to fibrous media

A) The boundary conditions

We have studied two cases corresponding to temperature and flux boundary conditions. For the first case, the temperatures at the medium boundaries are assumed to be known, corresponding to a system with guarded hot plates. The temperature of the front face is raised abruptly following a specific relation. The thermal boundary conditions are the following : the boundary surfaces at $x = 0$ and $x = E$ are both black surfaces kept at temperatures $f(t)$ and T_E respectively

$$T(0,t) = f(t) \text{ and } T(E,t) = T_E \quad \text{for } t > 0 \tag{11}$$

where f is a given function and $f(t) \rightarrow T_o$. $T(0,t)$ is the hot temperature, which increases very quickly versus time and reaches the constant value T_o. $T(E,t)$ is the cold temperature which remains constant with time. An electric resistance makes it possible to keep a high temperature T_o at the front face of the medium studied and a low temperature T_E at the other face. The contribution by Joule effect makes it possible to quantify the insulating character of the conductor. The radiative boundary conditions [4] are for $t \geq 0$

$$\begin{aligned} I_\lambda(0,\mu,t) &= I_{b,\lambda}(f(t)) \quad \text{for } 0 < \mu \leq 1 \\ I_\lambda(E,\mu,t) &= I_{b,\lambda}(T_E) \quad \text{for } -1 \leq \mu < 0 \end{aligned} \tag{12}$$

For the second case (flux boundary conditions), the two faces $x = 0$ and $x = E$ are in contact with ambient temperatures $T_{\infty,0}$ and $T_{\infty,E}$ respectively and the superficial exchanges between wall and ambient are characterised by convectif exchange coefficients h_o and h_E. Outside of two boundaries, we set two radiation sources of different intensity respectively, which vary versus time and defined by :

- $I_\lambda^+(\mu,t)$ with $0 < \mu \leq 1$ and $t > 0$ for the front face (i.e. in $x = 0$) ;
- $I_\lambda^-(\mu,t)$ with $-1 \leq \mu < 0$ and $t > 0$ for the back face (i.e. in $x = E$).

We studied insulating materials made of glass wool, composed of silica fibers. These fibers are randomly oriented in planes parallel to the boundaries of the medium. We consider the surface as being practically transparent, knowing that the density of fibers is low in the front and back plane. Thus, the probability for an incidental ray of meeting a fiber in the interface plane is negligible compared to make one's way through the air. In this case, the radiative boundary conditions are given by :

$$\begin{aligned} I_\lambda(0,\mu,t) &= I_\lambda^+(\mu,t) \text{ for } 0 < \mu \leq 1,\ t > 0 \\ I_\lambda(E,\mu,t) &= I_\lambda^-(\mu,t) \text{ for } -1 \leq \mu < 0,\ t > 0 \end{aligned} \tag{13}$$

and the thermal boundary conditions by :

$$\begin{aligned} &\text{in } x = 0\ ,\ -k(T)\frac{\partial T}{\partial x} + h_o(T)(T - T_{\infty,o}) = 0 \\ &\text{in } x = E\ ,\ \ k(T)\frac{\partial T}{\partial x} + h_E(T)(T - T_{\infty,E}) = 0 \end{aligned} \tag{14}$$

The convective exchange coefficients expressions h_o and h_E are determined from the Nusselt, Rayleigh, Prandtl and Grashof numbers [6]. For the two cases corresponding to the temperature and flux boundary conditions, the medium layer is initially at uniform temperature :

$$T(x,0) = T_E \quad \text{for all } 0 \leq x \leq E \tag{15}$$

B) Numerical solution of the coupled system of equations

The above equations define a strongly coupled system of partial and integro differential equations where the unknowns are the monochromatic radiation intensity and the temperature field. There is no known analytical solution to these equations. Then, the solution is obtained numerically using a discretization of the medium.

The RTE is solved using the method described in [11,12]. This method use a multi–flux model. An angular discretization technique (Discrete Ordinates approximation) is applied in order to express the RTE in an inhomogeneous system of linear differential equations associated with Dirichlet boundary conditions. This system is solved by a direct method (matrix exponential method), after diagonalizing the medium characteristic matrix, which made it possible to circumvent the numerical instability problem. This method is efficient in terms of computational times and the solution is analytical in space.

The energy equation is solved in space, by the finite element method P^2, using a non-uniform mesh. The resulting differential system in time is integrated using the implicit Runge–Kutta method adapted to stiff equations.

C) Simulation results

We studied a material composed of silica fibers with a diameter of 7 microns randomly oriented in planes parallel to the boundaries. It is a material close to those used in heat insulation. The thickness *E*, the density ρ and the specific heat capacity c_p of the fibrous medium are equal to $E = 10\,\text{cm}$, $\rho = 20\,\text{kg/m}^3$ and $c_p = 670\ \text{J/(kg K)}$, respectively. For the numerical application, 12 discrete polar directions and 211 wavelengths significant for the medium and varying from 2.5 to 25 μm are used. For the two problems; corresponding to the temperature and flux boundary conditions, we used a non–uniform spatial mesh and refined the zones which have strong temperature gradients. We used a very small time step $(\Delta t = 0{,}5\,s)$ for stability and accuracy.

For the first problem corresponding to the temperatures imposed on the boundaries, we calculated the temperature field and the heat fluxes in the medium according to time when $T_o = 400\,\text{K}$, $T_E = 300\,\text{K}$ and the temperature imposed f follows the law :

$$f(t) = \begin{cases} (T_o - T_E)\,t + T_E & 0 \le t \le 1\,s \\ T_o & \text{if } t \ge 1\,s \end{cases} \tag{16}$$

Figures 3, 4, 5 and 6 show the evolution of temperature and fluxes according to time and position in the medium. We point out that initially the temperature is constant and equal to T_E throughout the medium. At $t = 1\,s$, the imposed temperatures are equal to T_o at $x = 0$ and T_E at $x = E$. For low values of *t*, the material is at a temperature much lower than the black body located in $x = 0$ and the temperature gradient is very significant : radiative and conductive fluxes take high values. The radiation and conduction coupling leads to the existence of a maximum for radiative flux : at this point, the radiative energy source term is null. When the steady–state is reached $(t \cong 1250\,\text{s})$, the total heat flux becomes constant and equal to 47.8 W/m^2, a numerical value that we found from the previous developed steady–state model [12]. We also find the same values for the temperature field, the radiative and conductive fluxes. This validates the numerical method used to solve the coupled system of equations, in transient state.

For the simulation of the second problem corresponding to the flux boundary conditions, the medium is lightened by a black body, rising very quickly (1 s) from a temperature of 300 *K* to a temperature of 600 *K* on the front face (in $x = 0$) and remaining at a temperature of 300 *K* on the back face (in $x = E$). These two temperatures are then kept constant versus time. Thus, we can consider the axial symmetry in our model, since the radiative boundary conditions respect this symmetry. We suppose that the ambient temperature of the air on side of the front face of the medium is heated and follows the following linear evolution :

$$T_{\infty,0}(t) = \begin{cases} (150 \cdot \dfrac{t}{120} + 300)\,K & \text{when} \quad 0 \le t \le 120\,s \\ 450\,K & \text{when} \quad t \ge 120\,s \end{cases}$$

We assume that the ambient temperature of the air on side of the back face of the medium remains constant versus time and is equal to $T_{\infty,E}(t) = 300\,K$. On the figures 7, 8, 9 and 10, we represented the evolution of the temperature field and fluxes versus time and according to the position in the medium. On these four figures, the curves at $\mathrm{t_f} \cong 1500\,\mathrm{s}$ are related to the steady-state behaviour. The temperature reach a maximum in the medium : it is the loss by convection on the front face which is responsible. In particular, that leads to a conductive flux with negative value near to the front face. The peak of radiative flux is due to the fact that the incidental flux increases very quickly : the radiation is propagated in the medium at a speed close to that of the light in the vacuum. The elevation of temperature cannot be done at the same speed, therefore the conductive flux does not vary in the same way. The fast temperature variation on the front face leads to a conductive flux which vary also strongly. During first quarter of time necessary to reach the steady–state, the temperature field is practically linear. Then, the convergence towards the steady–state is much slower. When the system reaches the steady–state, the total heat flux becomes constant and equal to approximately 205 W/m^2, with a relative error between the abscissas very weak (lower than 0,6 % in our case).

In addition, we can say that the computing times for the first and the second programs are 54 h 57 min 40 s and 82 h 04 min 10 s respectively, on a Pentium II with the convergence criterion $\varepsilon = 10^{-6}$ relating to the relative variation of the temperature versus time. These two calculating times may seem to be long but they explain themselves by the complexity of the models. Moreover, with these application examples, the algorithms proved to be robust and stable.

Conclusion

The heat transfer by radiation and conduction in a fibrous media is studied with temperature or flux boundary conditions. The studied media are non-grey, anisotropically absorbing, emitting and scattering. The Mie theory is used to determine the effective section and the phase function of scattering. The thermal conductivity is temperature dependent. The RTE and energy equation are solved using a multi-flux model with a finite element method P^2. We calculated the heat transfer and the evolution of the temperature versus time in the medium. Our studies made possible the knowledge of the thermal behaviour of fibrous insulants. Moreover, we can determined accurately the role of several parameters such as : chemical composition, fiber diameter and fiber distribution inside the medium. The further development of this study will be related the validation of the model : the medium will be under strong heat fluxes on the front face.

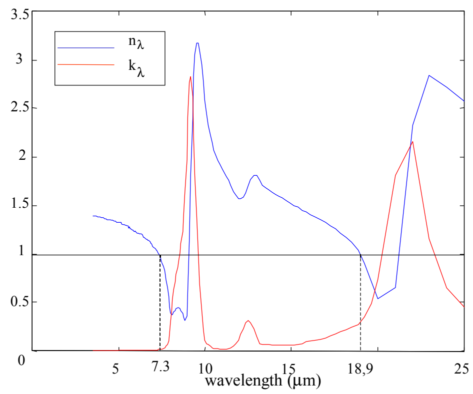

Figure 1. Variations of n_λ and k_λ of the silica.

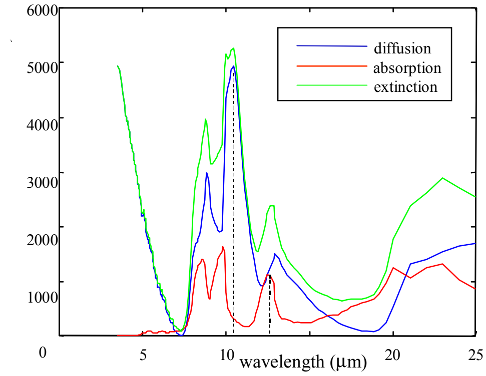

Figure 2. Effective section.

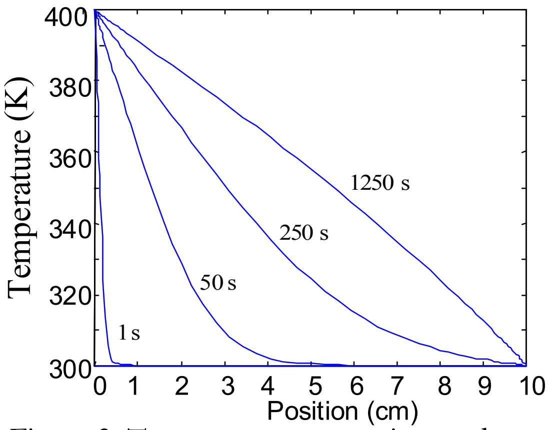

Figure 3. Temperature versus time and according to the position in the medium.

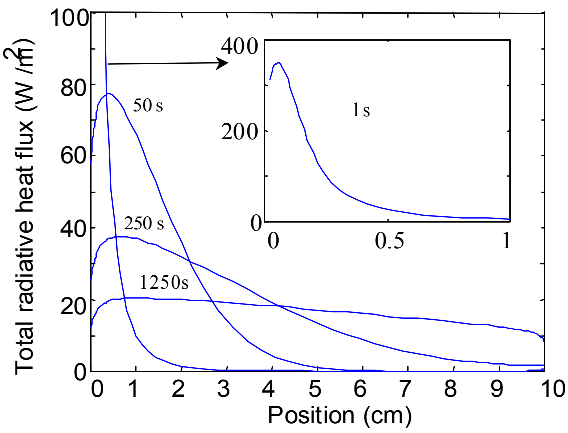

Figure 4. Total radiative heat flux versus time and according to the position in the medium.

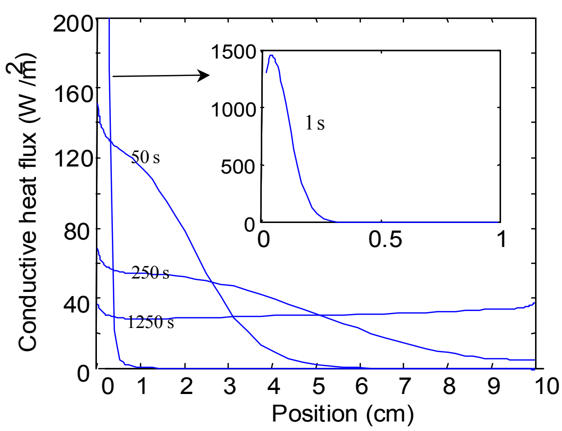

Figure 5. Conductive heat flux versus time and according to the position in the medium.

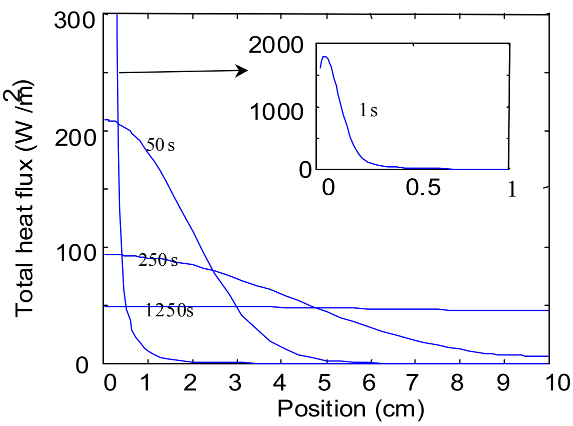

Figure 6. Total heat flux versus time and according to the position in the medium.

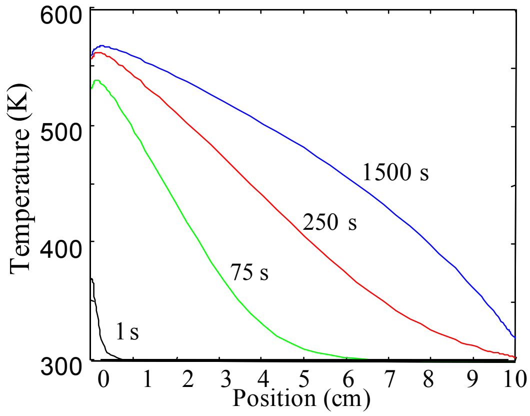

Figure 7. Temperature versus time and according to the position in the medium.

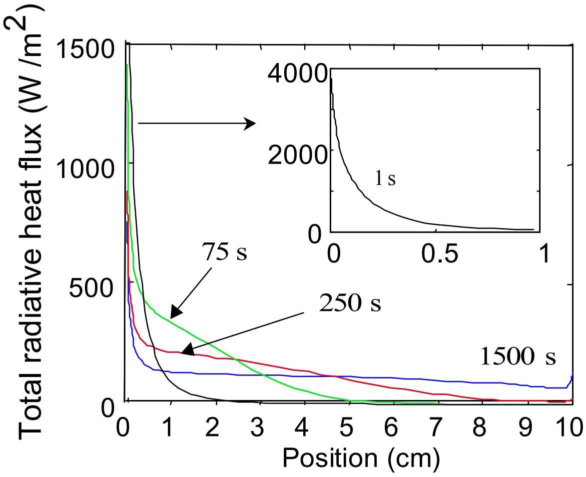

Figure 8. Total radiative heat flux versus time and according to the position in the medium.

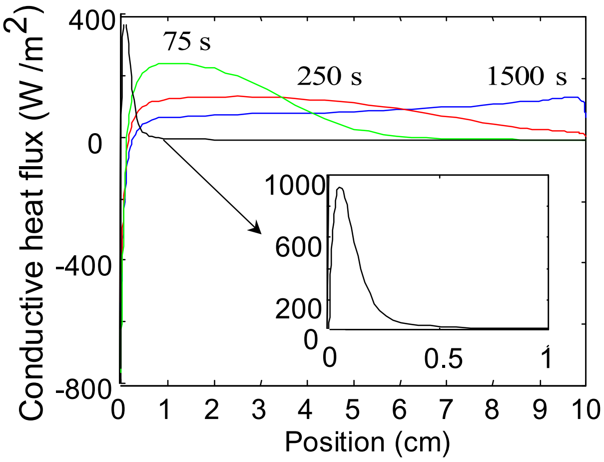

Figure 9. Conductive heat flux versus time and according to the position in the medium.

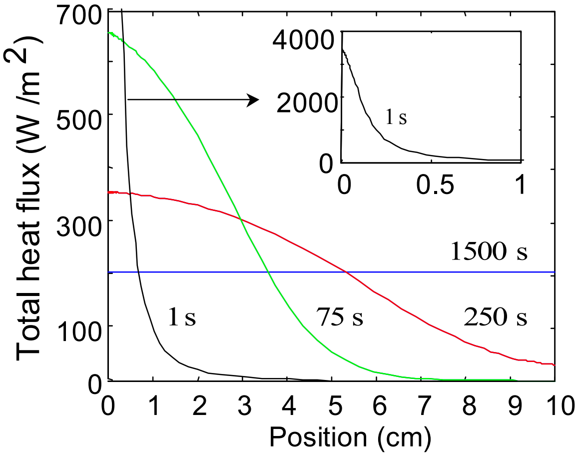

Figure 10. Total heat flux versus time and according to the position in the medium.

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