Some aspects to the validity of the discrete ordinate method for the radiation simulation in the glass tanks

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Glass melt is a semitransparent medium and radiation heat transport has a significant impact on the energy transport in the melt. Radiation in glass can be absorbed or passed through depending on the absorption coefficient and on glass tank dimensions. For silica-based glasses, typically 3 spectral bands can be distinguished for the wavelength dependent absorption coefficient. For window glass between 0.5 and 2.7µm, the absorption coefficient is low; above 2.7µm it becomes higher and the glass is nearly opaque. For large optical thickness, radiation is absorbed and emitted locally, thus according to literature, the Rosseland approximation is valid. Nevertheless, for intermediate or low optical thickness the radiation transfer equation shall be solved separately, and a more elaborate radiation model is needed. For glass melts, this model should be ideally spectral and valid for a large range of optical thicknesses. The Discrete Ordinate Method is one of the well-established radiation models and can be integrated into a CFD code. However, our verification on the TC21-RRT1 tank with κ=50m-1 × 1m shows significant differences between DOM and Rosseland. A bottom temperature difference of about 20K and significant differences in the recirculation intensity are stated. Using a 1D radiation problem with boundary conditions similar to the situation in glass tanks, this inconsistency can be explained. The results are significantly different between the Rosseland and DOM models. For large optical thickness, an exact solution for the radiation problem published by Heaslet and Warming [1965] fits well with the Rosseland approximation. One can also show theoretically that for large optical thickness ($\kappa > 10$), DOM is not valid. Because even clear glass melts present higher absorption coefficients for $\lambda > 2.7$ µm, standard multiband DOM is inappropriate as a radiation model of the melt in glass tanks.

Introduction

Radiation heat transfer plays a predominant role in the melting process of glass. Glass is a semitransparent media, part of the radiation can be readily absorbed or can pass through the glass on a long range. This behaviour depends on spectral absorption of the glass $a(\lambda)$ and on glass tank dimensions (typical glass tank depth L=1 m). The optical thickness ($\kappa = a \times L$) is usually used to appreciate that physics. Roughly, 3 spectral bands can be distinguished for the absorption coefficient of typical window glass. Between 0.5 µm and 2.7 µm the absorption is low and above it becomes stronger and nearly opaque.

Fig.1 presents spectral distribution of the absorption coefficient for different types of clear glass compared to the black body radiator [1].

For a mathematical model of temperature and advection of the melt in a glass tank, it is crucial to use an appropriate radiation model.

Mathematical Aspects

Usually glass is considered as an absorbing non-scattering medium. If the medium is assumed to be gray, radiation transfer equation becomes:

$$\frac{dI}{ds} = -aI + \frac{n^2 \cdot \sigma \cdot T^4}{\pi} \tag{1}$$

where I is the radiation intensity travelling in a single direction per unit solid angle and crossing a unit area normal to the direction of travel. From (1), the intensity distribution through the medium is obtained so that the net radiative energy source distribution can be found. This source is inserted into the energy conservation equation, to obtain the temperature distribution in the medium.

$$\vec{\nabla} \cdot \left(\rho \cdot C_p \cdot \vec{u} \cdot T\right) + \vec{\nabla} \cdot \left(-k \cdot \vec{\nabla} T\right) + \vec{\nabla} \cdot \vec{q}_r = 0 \tag{2}$$

Thus, to solve equation (2), an expression for the divergence of the radiant heat-flux vector ∇q_r is required. Many methods are known to solve this problem. The purpose of this paper is not to give a review of different models[2] but to compare and to check the validity of the Rosseland approximation and of the Discrete Ordinate Method (DOM) for an industrial application.

The Rosseland approximation requires that the medium is optically thick and the radiation intensity is isotropic in all angular directions. This condition is only respected for low temperature gradients. In the development of the heat flux vector[2], second order terms are truncated, and finally the divergence of the radiant heat-flux vector is given by Rosseland[3] in a 1D formulation:

$$\frac{dq_r}{dx} = -\frac{16}{3a} n^2 \sigma T^3 \frac{d^2T}{dx^2} \tag{3}$$

At the interfaces, because temperature gradients are strong, and the Rosseland approximation is not valid, a "Radiation slip" boundary condition shall be used. Temperature gradient and heat flux at the walls are corrected. For its simplicity, the Rosseland approximation is frequently used in glass tank simulations.

However, spectral aspects of the radiation heat transfer cannot be treated properly with this method and especially for clear glass, optical thickness can be small in the "optical window" of the absorption coefficient. The DOM is an extension of an approach called 2-flux method [2] . The DOM consists of discretizing the domain spatially and with respect to angle. The divergence of the radiant heat-flux vector is calculated from the solution of (1):

$$\vec{\nabla} . \vec{q}_r = a . \left[\int_{\Omega=0}^{4\pi} I . d\Omega - 4n^2 . \sigma . T^4 \right] \tag{4}$$

In a 1D case, with the 2-flux method the divergence of the radiant heat-flux vector is simplified to:

$$\frac{dq_r}{dx} = -\frac{4}{a} n^2 \sigma T^3 \frac{d^2T}{dx^2} \tag{5}$$

We can point out that, compared to (3) the factor 4/3 is absent in this relation. The 2-flux method is considered suitable for isotropic media[4]. The term (5) is then introduced as a source term in the energy equation. In numerical algorithm for the solution of (2) the source term is then linearised.

Validation test cases

The 1D case is an efficient tool to determine the validity of radiation modelling. The choosen boundary conditions are key parameters for identifying the correct result. Different cases may be proposed to validate radiation models. Table 1 shows the different possibilities. The most convenient case especially for large optical thickness is given in column 3. The other possibilities present certain disadvantages, some of them need very thorough precision in temperature calculation, or are not very representative for boundary condition used in glass melts, or exact solutions are not available. Results in column 3 test case used are represented in Fig.3. The absorption coefficient is chosen equal to 50 m^{-1}, heat flux is fixed to a value of 2000 w/m^2. The grid dimension is chosen smaller than the mean free path length[5] .

Discussion and conclusion

Fig 2 and 3 show a difference larger than 20% on bottom temperature (at L=1m) between DOM and Rosseland approximation. Furthermore, the Rosseland solution is similar to the exact one[6]. This difference is due to the methodology of the DOM.

For large optical thickness the coupling of radiation and energy equation is no longer correct with DOM. From the previous paragraph, this can be explained as follows. The source term in the energy equation is always linearised in numerical algorithms[7]. But for large optical thickness, this source term tends towards Rosseland approximation and is related to temperature for the 1D case by equation (3)[4]. Nevertheless the source term calculated directly from isotropic intensity is equal to the expression in equation (5). For a pure radiation problem, the relation between source term and temperature is used to find the temperature field. When the temperature is calculated with the energy conservation equation (2) and the calculated temperature value used in the radiation equation, an error occurs on the final solution of 4/3.

The error induced by DOM has a drastic consequence on the modeling of thermo-convection in glass tanks. To illustrate this problem, the TC21-RRT1 test case in 2D approach with a fixed batch heat loss[8] is used to compare Rosseland and DOM. An average bottom temperature difference of about 20K is found. DOM overestimates the bottom temperature. DOM is not valid for large optical thickness. Multiband DOM is not appropriate, even for very clear glass having absorption coefficients quite high for wavelength >2.7 µm for silica-based glasses. On the other hand, Rosseland approximation cannot be used for small optical thickness (very clear glass or small depth), and its simplicity is somewhat unsuited to our problem:

- Spectral aspect is approximated, the medium is considered gray;
- Boundary condition at the vicinity of semi-transparent is not well treated, usually the specular behaviour is not taken into account;
- The classical Rosseland approximation does not include any information about the geometry of the medium.

Because thermoconvection is directly related to radiation heat transfer in glass melt, radiation model should be ideally spectral and valid for large range optical thickness.

Table 1. Boundary condition possibilities for radiation benchmark case

B.C	Two temperatures	Two fluxes	Temperature and heat flux	Two convection conditions
Graphics	T1 $\varepsilon = 1$ T2 $\varepsilon = 1$	q1 $\varepsilon = 1$ -q1 $\varepsilon = 1$	T1 $\varepsilon = 1$ q1 $\varepsilon = 1$	h1 $\varepsilon = 1$ h2 $\varepsilon = 1$
Output	Temperature profile and heat flux	Temperature profile	Temperature profile	Temperature profile and heat flux
Numerical solution	High precision	Ill posed problem	–	High precision
Exact solution	Available	Not available	Available	Not available

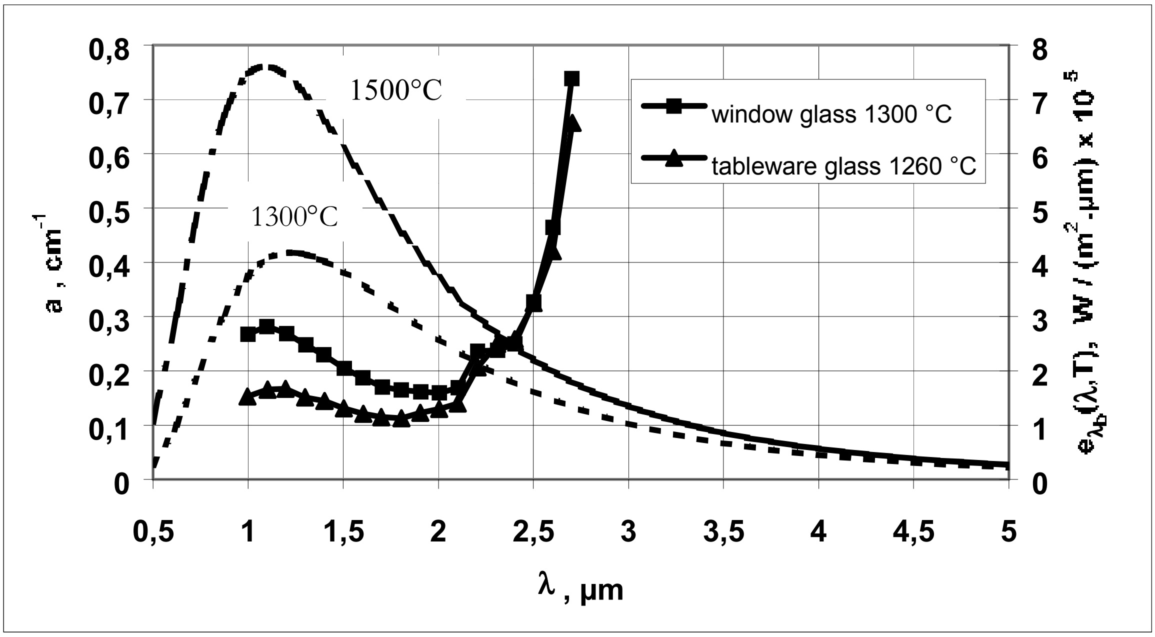

Fig.1: Wavelength dependence of absorption coefficient for window and tableware glass

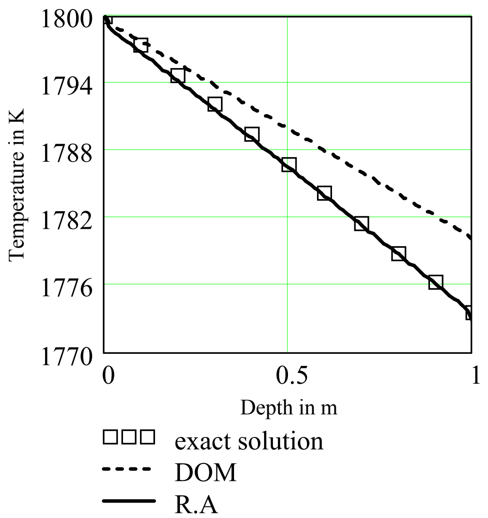

Fig.2: Temperature profile of Rosseland approximation and DOM for an absorption coefficient of 50 m^{-1} and q(x=0m) = -2000 W/m^2.
Only Rosseland solution fits with exact solution.

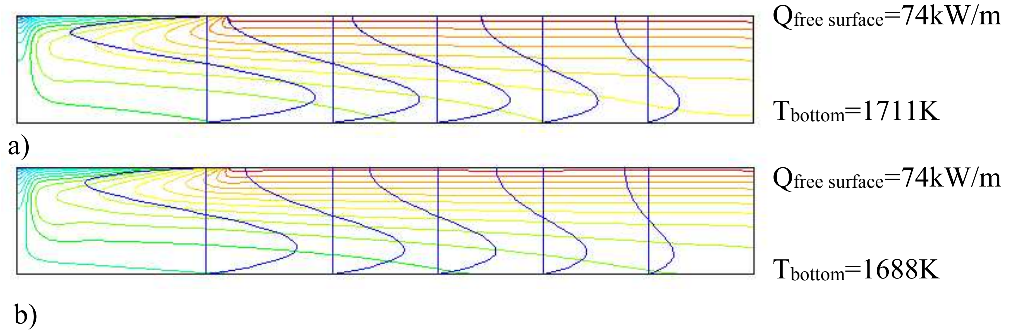

Fig.3: TC21-RRT1 test tank in 2D : Isotherms ($\Delta T = 10K$) and profiles of horizontal velocity for: a) DOM ; b)Rosseland Approximation .
Temperatures and flow are different in both models.

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