A physicist's approach of batch blanket modeling

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The glass batch is a mixture of raw materials. Through heating, the batch undergoes chemical reactions and converts into molten glass. Its role is primordial for the furnace thermodynamics, as a heat sink and as a thermal screen. A proper description of the convective motions in the bath requires the knowledge of its extension. We present a mathematical model where batch is represented as a homogeneous pure material that melts at a fixed temperature. The evolution of the upper and lower surfaces of the batch blanket is determined by numerical resolution of the thermal equation. We present a numerical method where the moving domain is replaced by a fixed one and compare the front velocities obtained by numerical simulation with analytical solutions.

Model

The batch blanket plays an essential role in the thermodynamics of the furnace since it acts as a heat sink and a thermal screen and its length determines the convective motions in the glass bath.

The glass batch is a mixture of raw materials. Upon heating, a series of chemical reactions occur and the mixture converts into molten glass during a complex process. In a simplified description we assume that all the chemical reactions take place at the same temperature and at an infinite reaction rate. This allows one to treat the glass batch as a homogeneous pure material which melts at a fixed temperature T_f. During the chemical reactions there is an important production of gases, mainly CO_2. We assume thermal equilibrium between CO_2 and the porous material of the batch. In addition, we use mean values for the density ρ, the thermal conductivity λ and the specific heat C_p of the ensemble in order to simplify the description of this complex porous material with percolating gases.

Under normal working conditions glass furnaces must be in a steady state regime. The pull rate of the furnace determines the value of the batch blanket velocity along the longitudinal *x* direction. This value is taken constant and equal to *u*. Hence, we assume a solid like type of motion.

The heat transfer inside the blanket can be made through conduction or trough convection by percolation of the gases. Both effects are of the same order of magnitude. Convection is important for the temperature profiles but has no effect on the length of the blanket. Neglecting heat transfer by convection leads to a much simpler system of equations, henceforth we use the approximation that the heat transfer is dominated by conductive phenomena. Since the blanket is much longer and larger than thicker, heat diffusion along the *xy* horizontal plane is neglected with respect to the vertical z direction. With these approximations the heat transfer in the blanket occurs trough conduction along the vertical direction. This leads to a two-dimensional model. A schematic view of batch is given in figure 1.

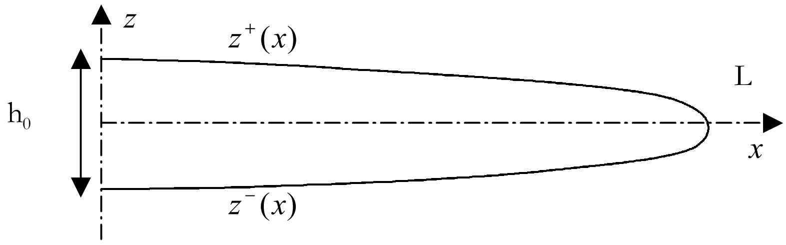

Fig. 1: Schematic view of the batch blanket

The glass batch enters into the furnace at $x = 0$ with an initial temperature T_0. The initial height of the blanket is h_0. We consider two steps during fusion. In a first step the batch material is heated until it reaches the temperature T_f. During a second step the batch converts into molten glass and the liquid-solid interface is kept at the fixed temperature $T = T_f$ during all the period of fusion. The evolution of the upper $z^+(x)$ and lower $z^-(x)$ surfaces of the glass blanket is determined by numerical resolution of the thermal equation

$$\rho\, C_p u \frac{\partial T}{\partial x} = \frac{\partial}{\partial z}\left(\lambda \frac{\partial T}{\partial z}\right). \qquad (1)$$

for $x \in [0, L]$, where L is the length of the blanket. The boundary conditions at $z^+(x)$ and $z^-(x)$ depend on the temperature values.

We have for the upper interface:

$$-\lambda \frac{\partial T}{\partial z}\bigg|_{z=z^-} = \varphi^-, \text{ if } T(z^-) < T_f, \qquad (2a)$$

$$-\lambda \frac{\partial T}{\partial z}\bigg|_{z=z^-} + \rho u \Delta H_f \frac{dz^-}{dx} = \varphi^-, \text{ if } T(z^-) = T_f. \qquad (2b)$$

And for the lower interface:

$$\lambda \frac{\partial T}{\partial z}\bigg|_{z=z^+} = \varphi^+, \text{ if } T(z^+) < T_f, \qquad (3a)$$

$$\lambda \frac{\partial T}{\partial z}\bigg|_{z=z^+} - \rho u \Delta H_f \frac{dz^+}{dx} = \varphi^+, \text{ if } T(z^+) = T_f. \qquad (3b)$$

ΔH_f is the sensible heat of fusion and φ^- and φ^+ are the heat fluxes coming from the glass bath and from the combustion chamber respectively. Equations (2a) and (3a) of continuity of fluxes at the lower and upper surfaces describe the heating of the batch material.

Equations (2b) and (3b) include also the energy needed for fusion of the batch when the temperature T_f is reached.

A dimensional analysis of the model provides a first estimate of the length L of the blanket,

$$L \approx \frac{\rho\, h_0\, u\left[C_p\left(T_f - T_0\right) + \Delta H_f\right]}{\overline{\varphi}^- + \overline{\varphi}^+}; \tag{4}$$

of the velocity of the liquid-solid interface

$$\frac{dz^\pm}{dx} \approx \frac{\mu\, \overline{\varphi}^\pm}{\rho u\left[C_p\left(T_f - T_0\right) + \Delta H_f\right]}; \tag{5}$$

and of the length of heating of the batch from the temperature T_0 until the temperature T_f

$$L_0^\pm \approx \frac{u\lambda\rho C_p \Delta T^2}{\overline{\varphi}^{\pm 2}}. \tag{6}$$

In the last expressions $\overline{\varphi}^-$ and $\overline{\varphi}^+$ are the mean values of the heat fluxes on the length L or $L_0^\pm$.

Numerical method

Equations 1-3 are solved numerically by finite volume methods. Using a first change of variable, $t = \frac{x}{u}$, the surfaces z^- and z^+ become functions of t and the spatial domain $\left[z^-(t), z^+(t)\right]$ also varies with t. To avoid this problem we can use another change of variable

$$\xi = \frac{z - z^-(t)}{z^+(t) - z^-(t)}. \tag{7}$$

that enables us to recover a fixed spatial domain, since ξ varies between [0,1] regardless of t.

Results and discussion

The values used to describe the physical properties of the glass batch are listed in table 1. They were taken from Fuhrmann[1].

Variable	*Value*	*Units*
ρ	1400	Kg/m³
C_p	1100	J/KgK
λ	$0.2465 + 1.88 \cdot 10^{-4} T + 1.69 \cdot 10^{-7} T^2$	W/mK
ΔH_f	$5.65 \cdot 10^5$	J/Kg
T_f	850	°C

Table 1- Properties of the glass batch (the temperature in the expression of λ is in Kelvin degree).

The glass batch has an initial temperature $T_0 = 30°C$ and its initial height is $h_0 = 10\ cm$. The velocity of the batch along the longitudinal direction is $u = 10^{-3} m/s$. We have first studied the case where the heat fluxes coming from the combustion chamber and from the glass bath are equal: $\varphi^- = \varphi^+ = 45 kW/m^2$. In a second case the heat flux coming from the combustion chamber is larger than the one coming from the bath: $\varphi^+ = 65 kW/m^2$ and $\varphi^- = 25 kW/m^2$. In both cases the total heat flux is $90 kW/m^2$. The results obtained numerically for the glass blanket profile are presented in figure 2.

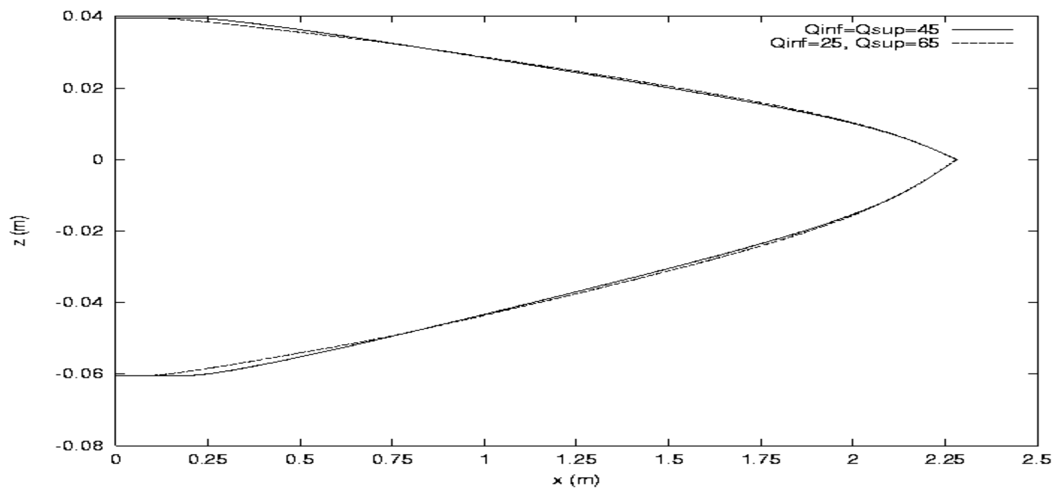

Figure 2- Batch blanket when i) $\varphi^- = \varphi^+ = 45 kW/m^2$ and ii) $\varphi^+ = 65 kW/m^2$ and $\varphi^- = 25 kW/m^2$.

The length of the blanket is the same in both cases L =2.28 m and this value compares well with the one obtained with the dimensional solution given by equation (4).

We restrict to the case where the upper and the lower heat fluxes are equal, to study the influence of the sensible heat of fusion on the shape of the blanket. We have taken in addition to the reference value of table 1, half of this value and twice of it. Results are presented in figure 3. One can see that the value of L increases with ΔH_f in agreement with equation (4).

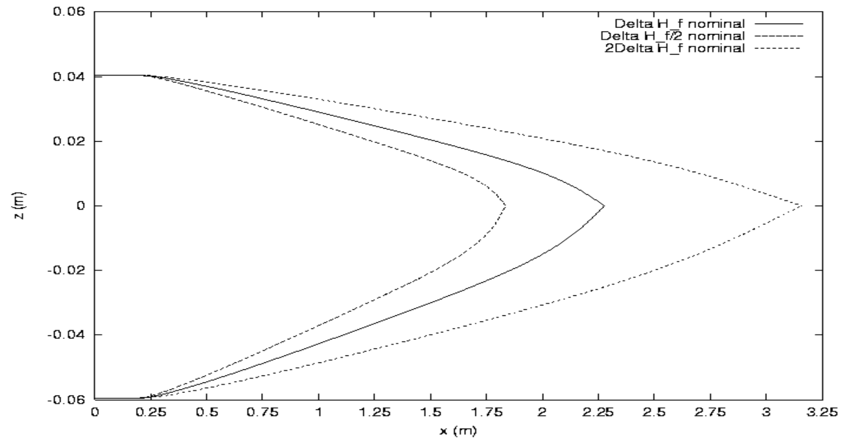

Figure 3- Batch blanket for tree different values of ΔH_f.

Finally we have changed the initial temperature of the bath. The results for 3 different temperatures are shown in figure 4. The length of the blanket decreases when T_0 increases.

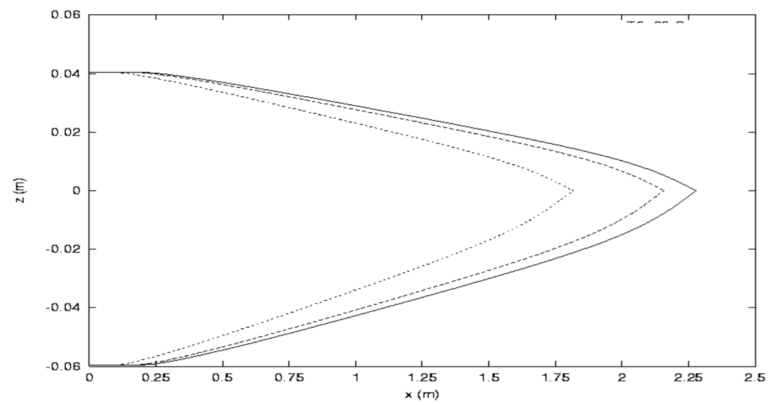

Figure 4- Influence of the initial temperature of the batch, T_0 in the blanket profile.

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[1] H. Fuhrmann, Glastechn. Ber. **46(11)**, p. 209 (1973).