Coupled radiative and conductive heat transfer in glass.

D. LACROIX, G. JEANDEL
LEMTA, U.H.P. Nancy1, B.P. 239, 54506 Vandoeuvre-les-Nancy cedex

The analysis of radiative heat transfer in semitransparent media is of interest for several research and industrial purposes such as material manufacturing and processing, combustion fabrication device, thermal insulation. In this study we only consider absorbing and non scattering media, which is the case for homogenous materials such as glass or thin coatings. In order to predict the radiative behavior of a semitransparent media we have to solve the radiative transfer equation (RTE). Moreover the absorption dependence with the wavelength of the radiation is taken into account for solving of the RTE.

For an homogeneous purely absorbing medium, we calculated the temperature field and heat fluxes of a material irradiated under a specific direction. Coupled radiative and conductive heat transfer were considered for a non grey semitransparent two-dimensional sample. Validation of the model and an original case for collimated radiation are also presented.

Analysis

Model description

The model we consider here is Cartesian two dimensional geometry where only absorption and emission are taken into account. Several kind of boundary conditions can be chosen for the simulation, such as transparent or opaque frontier with specular or diffuse reflection. Moreover, we are able to set a directional (or not) heat flux impinging a boundary. The figure 1 describes our model.

The medium properties change with wavelength (complex optical index) and temperature (thermal conductivity). In order to calculate temperature inside the semitransparent medium we have to solve two coupled equations which are the RTE (1) and the energy conservation equation(2).

$$(\vec{\Omega}\cdot\vec{\nabla})I_\lambda(\vec{s},\vec{\Omega}) = \kappa_\lambda\left[n_\lambda I_{b\lambda}(T) - I_\lambda(\vec{s},\vec{\Omega})\right] \quad (1)$$

$$\vec{\nabla}\cdot\left(\vec{Q}_r + \vec{Q}_c\right) = 0 \Leftrightarrow \vec{\nabla}\cdot\left(k_t(T)\vec{\nabla}T\right) = \int_{\lambda=0}^{\infty}\kappa_\lambda\left[4\pi n_\lambda'^2 I_{b\lambda}(T) - \int_{\Omega=4\pi} I_\lambda(\vec{s},\vec{\Omega})\,d\vec{\Omega}\right]d\lambda \quad (2)$$

The RTE equation can be solved using several numerical techniques. Most of them are well described in Modest[1] book. In this study we have used the Discrete Ordinate Method (DOM) which has been first proposed by Chandrasekhar and further developed by Lathrop[2] for heat transfer. This method is well known in radiative transfer simulation and has been improved over the last ten years. The energy conservation equation is solved using a finite difference method.

The computation of intensity field over the wavelength domain is obtained iteratively. Then we calculate the radiative source term to recover the temperature fields through the energy conservation. The mesh grid non-uniform in order to take into account the stiff change at the boundaries.

Radiative boundaries

As we said, the medium boundaries could be transparent or opaque. For a transparent boundary, we assume that the interface between the two media is planar and smooth. Then the reflection coefficient $\rho_\lambda(\theta)$ varies with the incidence angle θ and the optical properties of the two media. The reflection coefficient is governed by the following relation assuming $k^2 << n^2$ in the wavelength range we consider.

$$\rho_\lambda(\theta) = \frac{1}{2}\left(\frac{n_\lambda^2\cos\theta - n'_\lambda\sqrt{n_\lambda^2 - n_\lambda'^2\sin\theta}}{n_\lambda^2\cos\theta + n'_\lambda\sqrt{n_\lambda^2 - n_\lambda'^2\sin\theta}}\right)^2 + \frac{1}{2}\left(\frac{n'_\lambda\cos\theta - \sqrt{n_\lambda^2 - n_\lambda'^2\sin\theta}}{n'_\lambda\cos\theta + \sqrt{n_\lambda^2 - n_\lambda'^2\sin\theta}}\right)^2 \quad (3)$$

For that kind of boundary the intensity depends on the external temperature T_G, the reflected intensity and the intensity I_0 of the collimated radiation (if takes into account), it can be written as (4) :

$$I_\lambda(\vec{s},\vec{\Omega}) = \left(\frac{n'_\lambda}{n_\lambda}\right)^2\left[\tau_\lambda(\theta)I_{b\lambda}(T_G) + \tau_\lambda(\theta_0)f(\vec{\Omega},\vec{\Omega}_0)I_{0\lambda}(\vec{s},\vec{\Omega}_0)\right] + \rho_\lambda(\theta)I_\lambda(\vec{s},\vec{\Omega}') \quad (4)$$

where $\tau_\lambda(\theta)$ is the transmissivity for a given incidence angle and $f(\vec{\Omega},\vec{\Omega}_0)$ a normalized function used to described the $\vec{\Omega}_0$ as a combination of the Sn fixed directions around $\vec{\Omega}_0$.

For an opaque boundary, with a wall at the T_w temperature with a given emissivity $\varepsilon_{w\lambda}$, the intensity in the case of specular reflection we will be (5) :

$$I_\lambda(\vec{s},\vec{\Omega}) = \varepsilon_{w\lambda}n_\lambda'^2 I_{b\lambda}(T_w) + (1-\varepsilon_{w\lambda})I_\lambda(\vec{s},\vec{\Omega}') \quad (5)$$

Thermal boundary conditions

In order to calculate heat transfer within the media, energy equation must be solved. As in the case of radiative boundary conditions, we have to set thermal boundary conditions obtain by energy balance at the interface between the two media. Different kinds of boundary conditions can be assumed, however we consider two of them. One for the transparent boundary which exchange by means of free convection with the surrounding air. The second one describes the case of a thick wall at a given temperature far from the medium.

As an example if we consider the upper boundary of the medium exchanging with air the energy balance with collimated radiation leads to the following expression (6) :

$$\begin{aligned} & -k_t\left.\frac{\partial T}{\partial y}\right|_{y=y_{\max}} + \int_{\lambda=\lambda_{\lim}}^{\infty}\kappa_\lambda \pi I_{b\lambda}(T_G)\,d\lambda + \int_{\lambda=\lambda_{\lim}}^{\infty}\kappa_\lambda\Phi_{0\lambda}(x,y_{\max})\,d\lambda = \Lambda \\ & \Lambda = h\left[T(x,y_{\max}) - T_G\right] + \int_{\lambda=\lambda_{\lim}}^{\infty}\varepsilon_\lambda \pi I_{b\lambda}T(x,y_{\max})\,d\lambda \end{aligned} \quad (6)$$

We only consider wavelengths greater than λ_{lim}=5µm in the case of glass. Since it is an opaque materials for infrared radiation above 5µm. The measured infrared optical properties of glass will be given below. The convection heat exchange coefficient h is calculated using classical Nusselt correlation for free convection (above and under) a plate for a given local film temperature.

In the case of an opaque boundary such as those on the lateral face of the media, we can write the energy balance such as (7):

$$-k_t \left.\frac{\partial T}{\partial x}\right|_{x=0} + \int_0^{\lambda_{\lim}} \varepsilon_{w\lambda} \pi n_\lambda'^2 I_{b\lambda} T(0,y)\, d\lambda = K_g \left[T(0,y) - T_\infty\right] + \int_0^{\lambda_{\lim}} \kappa_{w\lambda} F_{x,\lambda}^{-}(0,y)\, d\lambda \quad (7)$$

where K_g is a global heat exchange coefficient per surface unit and $F_{x,\lambda}^{-}$ is the radiative heat flux in the negative x direction which can also be defined as (8) :

$$F_{x,\lambda}^{-} = \int_{\vec{\Omega}\cdot\vec{n}_w<0} I_\lambda\left(\vec{s},\vec{\Omega}\right)\left|\vec{\Omega}\cdot\vec{n}_w\right| d\vec{\Omega} \quad (8)$$

Discretization of the equations

The RTE equation necessitates two discretizations. The first one over a set of chosen radiative propagation directions. In the case of the S_n-DOM method, the quadrature sets are fixed. In our calculations we used S_8 (80 directions) and S_{12} (168 directions) for the sake of precision. In the case of two dimensional model, only half number of the quadrature are needed. The second discretisation is made on the differential term of the RTE considering a non uniform grid, which is established with a geometric decrement usually takes equal to 0.85.

This kind of discretization is well known for the DOM method and has been discussed several time[3]. Each rectangular cell face of the domain is marked with a label. The it is easy to calculate the intensity at the center of the cell using a combination of the faces intensities through a differencing scheme. Several kind of differencing scheme exist, we have used the step scheme and the variable weighted scheme which is well designed to minimize ray effect (also known as false scattering) that might affect calculations.

The discretization of the energy equation is not a simple problem since the thermal conductivity of the media depends on the temperature. Some new approaches using finite elements has been developed for the one dimensional radiative and conductive heat transfer in semitransparent media[4]. Nevertheless this technique doesn't exist yet for two dimensional medium. In our case, we will set the temperature field at the beginning of the calculation and use for the "i" iteration the "i-1" temperature field in order to calculate k_t.

Optical and thermal properties of the medium

In order to test our model we needed a semitransparent material, silica or float glass is an interesting media for this application. Our research team is specialized in visible and infrared spectroscopy. Complex index of the material can be retrieve from reflection and transmission on bulk material combined with Kramers-Kronig transformation. Typical values of n and k are given on the figure 2. We can easily see that the medium is non grey.

For more realistic computation we will have to take into account the evolution on the complex index with temperature. Glass radiative properties at high temperatures has been recently investigated by Van Nijnatten[5] et al.

Concerning the heat conduction coefficient k_t, a large set of experimental data is available. Several measurements in the range of room temperature till 1500K have been achieved. The following values of k_t (9) have been obtained for float glass, with T in Celsius degree, by Banner[6], Mann[7] and Andre[8].

$$\begin{cases} k_t = 1{,}1 + 1{,}29 \cdot 10^{-3} T : D.\,Banner \\ k_t = 1{,}14 + 6{,}24 \cdot 10^{-4} T : D.\,Mann \\ k_t = 0{,}995 + 8{,}58 \cdot 10^{-4} T : S.\,Andre \end{cases} \tag{9}$$

Results and discussion

Test case

In order to check our model we compared our results with those of Lee[9] et al. Who have recently published paper about radiative heating of molten glass in industrial furnaces. The test case is the following one figure 3. The dimension of the medium are : W=0.2 m, H=0.1m. The material is heated by hot gases (T_G=1800K) at the upper face which is transparent. The other boundaries are assumed to be black thick walls with temperature far from the medium equal to $T_\infty = 300K$. The grid size is 50×50 , and we used a S_8 angular discretization. For the spectral discretization we used 25 bands and we take the D. Mann correlation for thermal conductivity.

The results are almost the same. There is a slight variation that may be due to the difference the sets of optical properties. Another point that should be consider is the difference existing between the DOM method and the Rosseland approximation.

Heating with variable incidence

Here we are going to deal with lightning of the same glass medium by a collimated external blackbody at 1000K.The surrounding gas and the walls are at the room temperature (300K) According to the Wien's law the maximal intensity occurs at 2.9μm, the resulting temperature field in the medium is colder. The collimated radiation impinges the upper part of the media with an incidence angle equal to : 0°, 30°, 45° and 60°. The results are given on figure 4 and figure 5.

One can easily that temperature field are quite as we would expect. The hottest one is obtained for normal incident radiation where isothermal lines are parallel to the upper face. For 30° incidence angle the maximum temperature is shifted to the left part of the glass sample according to the direction of the collimated radiation. The temperature level in the lower part of the medium is almost the same since conduction is the main heat transfer mode in this area. For higher incidence angle 45° and 60° the reflection occurring at the north boundary become non neglictible and there is less radiant energy getting inside the glass sample. In these last cases there is essentially conduction. This remarks can also been achieved from the heat fluxes observation.

Conclusion

The model presented here is able to solve radiative and conductive heat transfer in a semitransparent media. Several points concerning the numerical resolution have been investigated. We show that DOM method for radiative heat transfer gave better results than the Rosseland approximation. Concerning the collimated radiation above such sample we have shown that the temperature fields are strongly affected by the incidence angle of the impinging radiation. Other points which have not been discussed here have been studied, such as the influence of the differencing scheme, the effect of the spatial and the angular discretization modification on the computed results. They have been discussed in a more detail work we have been accepted for publication. Further works will have to take into account scattering inside inhomogeneous media, in order to consider more media.

Figures

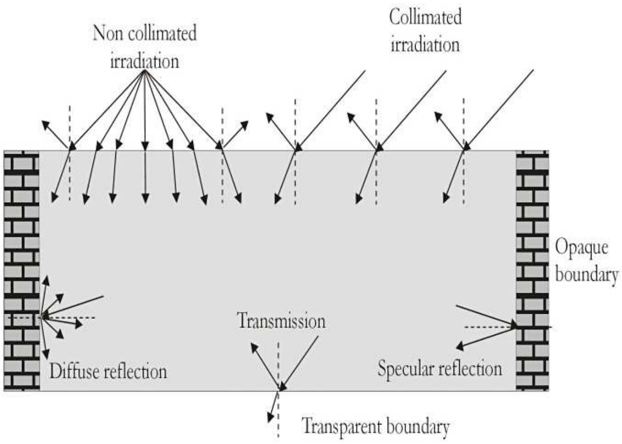


Figure 1

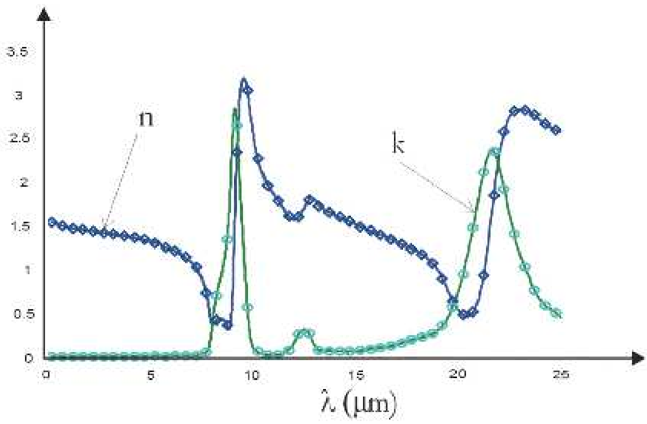
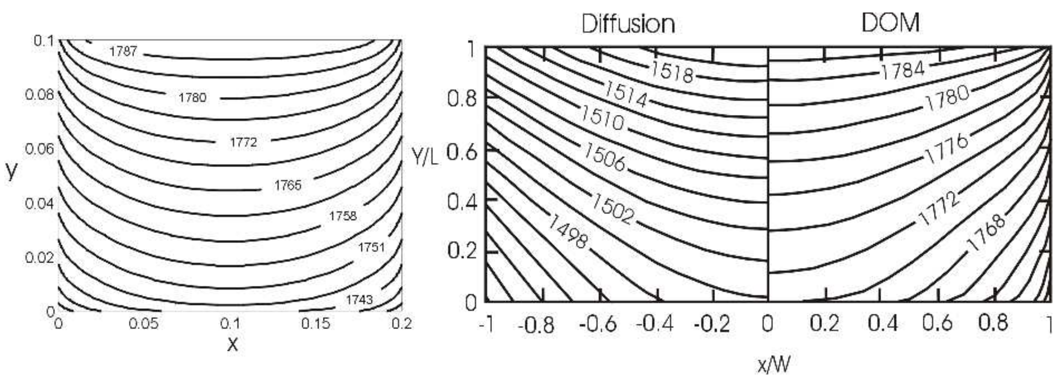
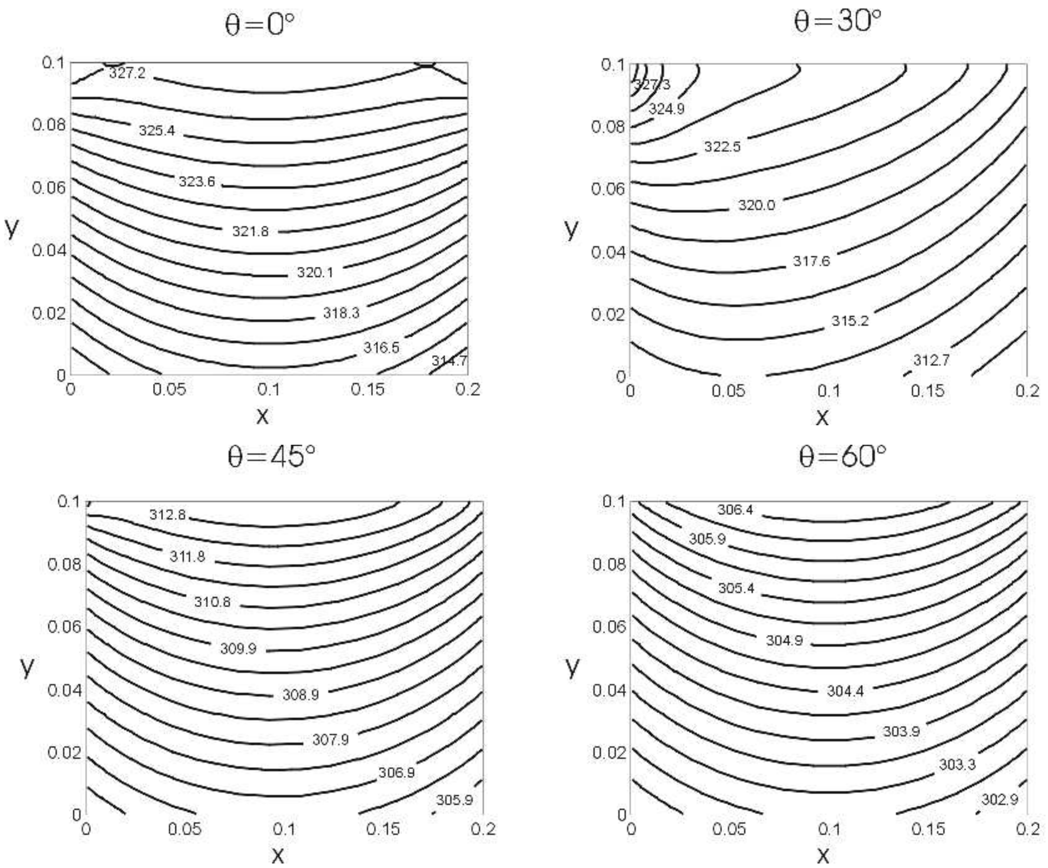


Figure 2



Figures 3



Figures 4 and Figure 5

---

[1] M.F. Modest, in *Radiative Heat Transfer*, edited by Mc Graw Hill International Editions (1993).

[2] K.D. Lathrop, J. Comput. Phys. **4**, p. 475-498 (1969).

[3] S.T. Thynell, Int. J. Engineering Science **36**, p. 1651-1675 (1998).

[4] F. Asllanaj, J.R. Roche and G. Jeandel, Int. J. Numerical Method Heat Fluid Flow **11**, p. 449-472 (2001).

[5] P.A. Nijnatten, A.J. Faber and J.T. Brokehuijese, Ceram. Eng. Sci. Proc. **20**, p. 47-56 (1999).

[6] D. Banner, Ecole Centrale Paris - St Gobain Recherche, Propriétés radiatives des verres et des fontes de silicates. Modélisation des transferts de chaleur (1990).

[7] D. Mann, R.E. Field and R. Viskanta, Wärme und Stoffübertragung **27**, p. 225-231 (1992).

[8] S. Andre, LEMTA - INPL, Identification de la diffusivité phonique du verre par méthode flash de 20°C à 600°C (1992).

[9] K.H. Lee and R. Viskanta, Glastech. Ber. Glass Sci. Technol. **72**, p. 254-265 (1999).